Non-pool based line planning

Optimization in public transport

Irene Heinrich, Philine Schiewe, and Constantin Seebach

Logistics in Operations Research, Helsinki 2023

Institute of Mathematics
TU Darmstadt, Germany

9th of May 2023

ERC project EngageS at TU Darmstadt

EngageS: Next Generation Algorithms for Grabbing and Exploiting Symmetry

- all areas of CS, mathematics, OR: intrinsic symmetries
- detect & exploit symmetry algorithmically,
- bring theory and practice closer together

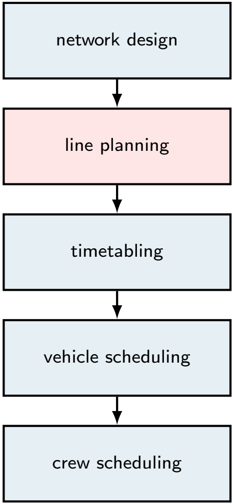
TECHNISCHE UNIVERSITÄT DARMSTADT

TU Darmstadt, Hesse, Germany

European Research Council

EngageS Project

Public transport planning: a sequential process



Operator:

- minimize costs
- maximize robustness

Passengers:

- minimize perceived travel time
- maximize convenience
- minimize delays

Line planning: literature

- heuristics for bus transit networks
  [Wan and Lo, 2003, Kepaptsoglou and Karlaftis, 2009, Farahani et al., 2013, Arbex and da Cunha, 2015, Cancela et al., 2015]
- maximizing direct travelers/minimizing perceived travel time
  [Bussieck et al., 1997, Schöbel and Scholl, 2006, Goerigk and Schmidt, 2017, Bull et al., 2018]
- minimizing costs
  [Claessens et al., 1998, Şahin et al., 2020, Torres et al., 2008, Torres et al., 2011]
- line planning while generating lines
  [Borndörfer et al., 2007, Torres et al., 2011, Gattermann et al., 2017, Borndörfer et al., 2018, Pätzold et al., 2018, Masing et al., 2022]

Cost-minimal line planning on all lines (LPAL)

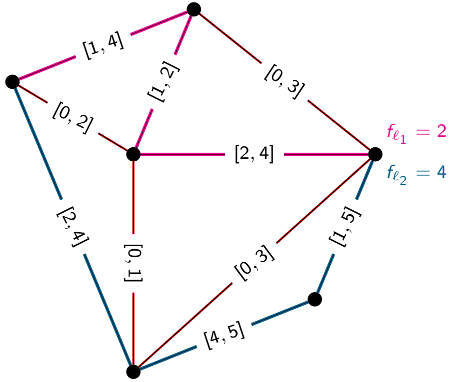
- public transport network (PTN)
- line: simple path in PTN
- line concept $(\mathcal{L}, f)$ such that
  - $\rightarrow$ frequency constraints satisfied

$$f_e^{\min} \leq \underbrace{F_e^{(\mathcal{L},f)}}_{=\sum_{\ell \in \mathcal{L}:\ e \in E(\ell)} f_\ell} \leq f_e^{\max}$$

  - $\rightarrow$ costs are minimized

$$\text{cost}((\mathcal{L}, f)) = d_{\text{fix}} \cdot |\mathcal{L}| + \sum_{\ell \in \mathcal{L}} f_\ell \cdot \text{cost}_\ell$$

$$\text{cost}_\ell = c_{\text{fix}} + \sum_{e \in E(\ell)} c_e$$



Complexity results

class	only frequency-dependent costs ($d_{\text{fix}} = 0$)	with frequency-independent costs ($d_{\text{fix}} > 0$)
stars	polynomial ([3])	NP-hard ([3])
paths	polynomial for $f^{\max} \equiv \infty$ ([1])	NP-hard ([3])
trees	pseudo-polynomial ([3]) polynomial for $f^{\min} = f^{\max}$ ([3])	NP-hard ([3])
planar	NP-hard, even for $\{0, 1\}$ input ([3])	NP-hard, even for $\{0, 1\}$ input ([3])
general	NP-hard, even for $\{0, 1\}$ input ([2])	NP-hard, even for $\{0, 1\}$ input ([2])

[1] P. Gattermann, (2015), Generating Line-Pools, Master's thesis, Georg-August-Universität Göttingen

[2] P. Gattermann, J. Harbering, and A. Schöbel, (2017), Line pool generation, Public Transport, 9(1-2):7–32

[3] I. Heinrich, P. Schiewe, C. Seebach, (2022), Algorithms and Hardness for Non-Pool-Based Line Planning, ATMOS22

Paths & frequency-independent costs

Theorem

LPAL *is NP-hard, even if* G *is a path and* $f^{\min} = f^{\max}$ *or* $f^{\max} \equiv \infty$.

3-PARTITION

Input:

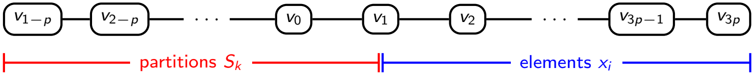
- $S = \{x_1, \ldots, x_{3p}\}$,
- $x_i \in \mathbb{N}_{>0}$

Find:

- $S_1 \sqcup \ldots \sqcup S_p = S$,
- $\sum S_k = h = \sum S/p$

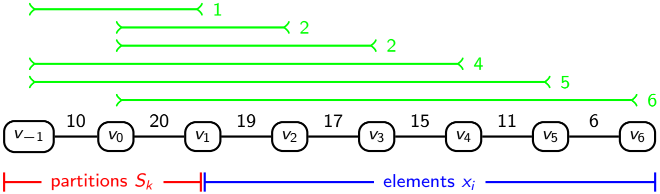
decision version of LPAL

$d_{\text{fix}} = 1,\ c_{\text{fix}} = 0,\ c \equiv 0,\ f^{\min} = f^{\max},\ K = 3p$



$$f^{\min}_{\{v_i, v_{i+1}\}} = \begin{cases} (p - i) \cdot h & i \leq 0 \quad \text{(increasing)} \\ \sum_{j=i+1}^{3p} x_j & i > 0 \quad \text{(decreasing)} \end{cases}$$

Paths & frequency-independent costs

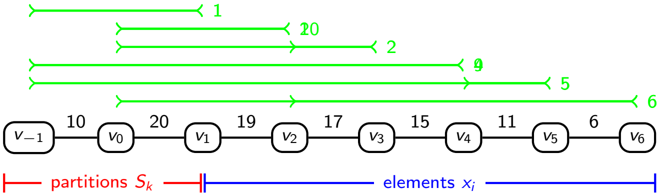


$S = \{1, 2, 2, 4, 5, 6\}$,
$p = 2,\ h = 10$
$S_1 = \{1, 4, 5\},\ S_2 = \{2, 2, 6\}$

For feasible partition construct $(\mathcal{L}, f)$ feasible with $\text{cost}(\mathcal{L}, f) \leq K$:

- for $x_i \in S_k$ construct line $\ell_{i,k} = (v_{k-p}, \ldots, v_i)$ with $f_{\ell_{i,k}} = x_i$
  - $S_1 \sqcup \ldots \sqcup S_p = S \Rightarrow |\mathcal{L}| = 3p = K$
  - $\sum_{\substack{\ell \in \mathcal{L}: \\ \text{left end at } v_{k-p}}} f_\ell = \sum_{i \in S_k} f_{\ell_{i,k}} = \sum S_k = h$
  - for $k \in \{1-p, \ldots, 0\}$: $F^{(\mathcal{L},f)}_{\{v_k, v_{k+1}\}} = \sum_{\substack{\ell \in \mathcal{L}: \\ \text{left end at } v_i, i \leq k}} f_\ell = (p-k)h = f^{\min}_{\{v_k, v_{k+1}\}}$
  - for $j \in \{1, \ldots, 3p\}$: $F^{(\mathcal{L},f)}_{\{v_j, v_{j+1}\}} = \sum_{\substack{\ell \in \mathcal{L}: \\ \text{right end at } v_i, i \geq k}} f_\ell = \sum_{i=j+1}^{3p} x_i = f^{\min}_{\{v_j, v_{j+1}\}}$

Paths & frequency-independent costs



$S = \{1, 2, 2, 4, 5, 6\}$,
$p = 2,\ h = 10$
$S_1 = \{1, 4, 5\},\ S_2 = \{2, 2, 6\}$

For $(\mathcal{L}, f)$ feasible with $\text{cost}(\mathcal{L}, f) \leq K$ construct partitions $S_1, \ldots, S_p$:

⚠ lines can have left and right end in $\{v_1, \ldots, v_{3p}\}$

- total frequency of lines with left end at v_i, $i \in \{1-p, \ldots, 0\}$, is h
- line ℓ_i with right end at v_i, $i \in \{1, \ldots, 3p\}$, satisfies

$$f_{\ell_i} = x_i + \sum_{j=i}^{3p} \delta_{\ell_j, v_i}^{\text{left end}} \cdot f_{\ell_j}$$

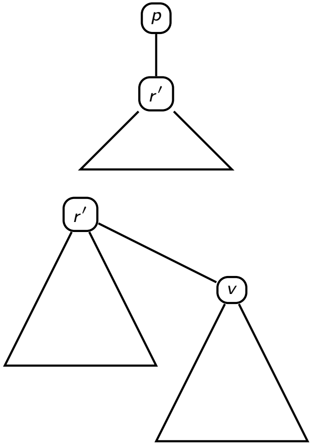
- transform solution in linear time

Trees with $d_{\text{fix}} = 0$ and bounded $f^{\max}$

- $f_e^{\max} \leq b$ for all $e \in E$
- construct rooted tree from leafs by
  - introducing a parent: $(T', r') \to (T' + p, p)$ if p is the only neighbor of r' in $V(T) \setminus V(T')$
  - merging two trees (T_1, r'), (T_2, r') if r' has only one child v in T_1 or T_2

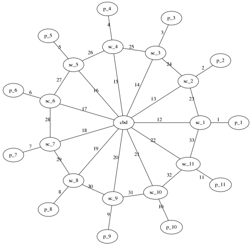
$\Rightarrow$ at most b lines end at r'

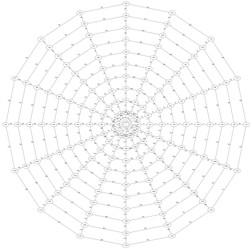
- dynamic program:
  - optimal objective value in $\mathcal{O}(nb^3)$
  - optimal solution in $\mathcal{O}(n^3 b^3)$



Current and future work

- exact polynomial-time algorithm for trees
- exact polynomial-time algorithm for cycles
- (parametrized) algorithms for parametric city and ring graphs
- (parametrized) algorithm for bounded treewidth
- including light spanners and optimal requirement graphs into our model





kiitos